

NY Workshop on the Symmetric Group

SCHEDULE AND ABSTRACTS

All talks take place in Room C198 of the CUNY Graduate Center.

Thursday, September 8, 2011 (Session chaired by Alexander Ryba)

9:45–10:00 WELCOME AND INTRODUCTORY REMARKS

10:00–10:30 **Speaker:** Harald Ellers (Allegheny College)

Title: Representation Theory of Centralizer Algebras

Abstract: Let p be a prime number and assume that (R, F, k) is a p -modular system; this means that R is a complete discrete valuation ring, F is the field of fractions of R , and k is the residue field $R/J(R)$ of characteristic p .

Let S_n be the symmetric group on n letters. If $l \leq n$, identify S_l with the subgroup of S_n consisting of all elements σ such that $\sigma(i) = i$ whenever $i > l$. The associated *centralizer algebra* $RS_n^{S_l}$ is defined to be the set of all elements a in the group algebra RS_n such that $\tau a = a\tau$ for all $\tau \in S_l$.

John Murray and the speaker are engaged in a project to try to understand the representation theory of the algebras $FS_n^{S_l}$, $RS_n^{S_l}$, and $kS_n^{S_l}$. The simple $FS_n^{S_l}$ -modules are known. Among the problems we would like to solve are the following.

- (1) Find all simple $kS_n^{S_l}$ -modules.
- (2) Find the blocks of $RS_n^{S_l}$ (and thus of $kS_n^{S_l}$).
- (3) Obtain information about the decomposition matrices of $RS_n^{S_l}$.
- (4) Relate information about blocks and simple modules for $kS_n^{S_l}$ to p -local information, along the lines of Brauer's First Main Theorem or Alperin's weight conjecture.

We have solved these problems only in special cases.

We will discuss recent progress. The degenerate affine Hecke algebras \mathcal{H}_{n-l}^k play an important role.

10:35–11:05 **Speaker:** Rosa Orellana (Dartmouth College)

Title: Stability of Clebsh-Gordan coefficients

Abstract: In general, the problem of decomposing the tensor product of two irreducible representations is called the Clebsh-Gordan problem. The quintessential example of stability is the Littlewood-Richardson rule, which answers the Clebsh-Gordan problem for the special linear group. In this talk I will discuss the stability of the Kronecker coefficients the Clebsh-Gordan problem corresponding to the symmetric group. In particular, I plan to discuss types of stability as well as bounds for the stability. This is joint work with E. Briand and M. Rosas.

11:10–11:40 BREAK

11:40–12:10 **Speaker:** Yaacov Kopeliovich (MEAG NY)

Title: Representations of Symmetric groups and Jacobians of cyclic covers of the \mathbb{CP}^1

Abstract: Let X be an algebraic with the equation

$$y^N = \prod_{i=1}^m (x - \lambda_i)^{R_i},$$

$\sum_{i=1}^m R_i = 0 \pmod N$ and let $Jac(X)$ be its Jacobian. For $N = 2$ Thomae discovered a remarkable formula that expresses the values of theta functions on points of order 2 associated with such Jacobian via certain polynomials associated with partitions of the set λ_i . Following the works of Bershadsky and Radul, and Nakayashiki, this result was generalized recently for any cyclic cover.

For $N = 2, 3$ the polynomials produce irreducible representations of S_m . This observation leads to the construction of basis and dimension calculation of space of the associated with fourth and third powers of theta functions via the hook formula. if $N = 2$ these are precisely the Frobenius relations. For $N > 3$ the polynomials can be associated with a class of representations of the Symmetric group, that generalize the construction of irreducible representations associated with partitions of k via Specht polynomials. I will explain this construction for any Symmetric group on l symbols. (this involves partitions of $m \geq l$ with no more than l columns) The exact dimension of these representations, their decomposition into irreducibles of S_l and basis aren't known to me. I hope they are known to the experts in the audience with obvious generalizations to theta functions.

12:15–12:45 **Speaker:** Artem Dudko (University of Toronto)

Title: On characters of inductive limits of symmetric groups

Abstract: In this talk we consider inductive limits of symmetric groups under block diagonal embeddings. Each such group defines an infinite Bratteli diagram that encodes the embedding scheme. The group G acts on the space X of infinite paths of the associated Bratteli diagram by changing initial edges of paths. Assuming that the number of ergodic measures for the system (X, G) is finite and the group G is simple, we establish that each indecomposable character $\chi : G \rightarrow \mathbb{C}$ is uniquely defined by the formula $\chi(g) = \mu_1(Fix(g))^{\alpha_1} \cdots \mu_k(Fix(g))^{\alpha_k}$, where μ_1, \dots, μ_k are G -ergodic measures, $Fix(g) = \{x \in X : gx = x\}$, and $\alpha_1, \dots, \alpha_k \in \{0, 1, \dots, \infty\}$. The talk is based on a joint work with Konstantin Medynets, Ohio State University.

12:50–14:15 LUNCH

14:15–14:45 **Speaker:** James Sellers (Penn State University)

Title: Arithmetic Properties for t -Core Partitions

Abstract: For a fixed positive integer t , we say that a partition of the integer n is a t -core if none of the hook lengths of the corresponding Ferrers diagram of the partition is divisible by t . Such t -core partitions are of immense interest in representation theory for numerous reasons. Moreover, the functions which enumerate the t -core partitions of n (for fixed t) have also been studied in their own right, and the arithmetic and combinatorial properties of these functions have proven to be intriguing. It is from this latter point of view that we will discuss the recent history of t -core partitions, focusing primarily on Ramanujan-like congruences which their enumerating functions satisfy. We will close by discussing a set of unexpected parity results satisfied by a specific family of t -core partition functions which are closely related to a family of partitions defined by George Andrews and Peter Paule in 2007.

14:50–15:20 **Speaker:** Jean-Baptiste Gramain (University of Paris)

Title: Hooks and bars in partitions

Abstract: It is a classical result that the partitions of an integer n label the irreducible complex characters of the symmetric group S_n . The partitions of n into distinct parts (also known as bar-partitions) label the so-called spin characters of the covering groups of S_n ,

which correspond to the projective representations of S_n . The combinatorics of partitions and bar-partitions then provide a powerful tool to study the representation theory of these groups. In particular, the degree of a character is given by the famous Hook-Length Formula, or its bar-analogue, the Bar-Formula, which involve the hook-lengths and bar-lengths in (bar-)partitions. Also, if p is a prime, then the p -blocks of these groups are parametrized by the p -cores and p -bar-cores of partitions. In this talk, I will present recent results about hooks and hook-lengths in partitions (in collaboration with C. Bessenrodt and J. B. Olsson) and about bars and bar-lengths in bar-partitions (in collaboration with J. B. Olsson). We give a complete description of the multiset of hook-lengths (or bar lengths) in a partition involving those in its p -(bar-)core and those in its p -(bar-)quotient. In particular, we derive some relative degree formulae for the degrees of (spin-)characters.

15:25–16:00 BREAK

16:00–16:30 **Speaker:** Dave Hemmer (SUNY Buffalo)

Title: Low Degree Cohomology For Symmetric Group Modules

Abstract: We discuss the state of the art in computing cohomology in low degrees for natural choices of symmetric group modules, including Young and Specht modules and irreducible modules. Some of the work is joint with F. Cohen and D. Nakano. Techniques from algebraic groups and algebraic topology, as well as symmetric group computations will play a role.

16:35–17:05 **Speaker:** Matt Fayers (University of London)

Title: The Mullineux map

Abstract: By tensoring an irreducible representation of the symmetric group with the sign representation, we obtain a new irreducible; since the irreducible representations are labelled by certain partitions of n , one obtains an involution on these partitions. In characteristic zero, where the irreducible representations are labelled by all partitions of n , this map on partitions is just conjugation. But in characteristic p , the irreducible representations are labelled by p -regular partitions of n , and the map on partitions (the “Mullineux map”) is harder to describe. We’ll describe this combinatorial map and explore some of its properties, including its relationship to James’s regularisation map.

Friday, September 8, 2011 (Session chaired by Dave Hemmer)

10:00–10:30 **Speaker:** Karl Mahlburg (Princeton University)

Title: Coefficient Asymptotic for Kac-Wakimoto characters

Abstract: In Kac and Peterson’s study of characters for affine Lie algebras, they proved a number of “Denominator identities” that related the weight multiplicities of irreducible submodules to theta functions. They then used modular inversion formulas and Tauberian theorems in order to derive asymptotics for these weight multiplicities; one of the simplest examples of affine Lie algebras leads to Hardy and Ramanujan’s famous formula for the asymptotics of $p(n)$, the integer partition function.

In this talk I will present joint work with K. Bringmann on the characters for affine Lie superalgebras that were later introduced by Kac and Wakimoto. In this setting, the characters are products of theta functions and Appell-type sums, which have recently been studied using developments in the theory of mock modular forms and harmonic Maass

forms. We find asymptotic series expansions for the coefficients of the characters with polynomial error.

10:35–11:05 **Speaker:** Amanda Folsom (Yale University)

Title: Mock modular forms and characters of Kac-Wakimoto

Abstract: Kac and Wakimoto recently established certain character formulas arising from affine Lie superalgebras. In this talk, we will discuss the “modularity” of these characters. In particular, we will discuss works of the author and Bringmann-Ono, which show that these characters may be realized as parts of certain non-holomorphic modular functions. Moreover, we show how these characters specialize to Ramanujan’s original “mock theta functions”, certain combinatorial q -series. As an application, we will then discuss joint work with Bringmann, which shows how the “modularity” of these characters can be exploited to obtain improved information regarding their asymptotic behaviors.

11:10–11:40 BREAK

11:40–12:10 **Speaker:** JP Cossey (University of Akron)

Title: Alperin weights of the symmetric group and combinatorics of partitions

Abstract: It has long been known that the ordinary irreducible representations of the symmetric groups are indexed by partitions. Recently, there have been some developments connecting certain classes of partitions to different characteristic p objects in the symmetric group, including the p -regular conjugacy classes, the Brauer characters, and the Alperin weights. In the talk we will discuss the “natural” class of partitions corresponding to each object, and look at some related combinatorial and algebraic questions.

12:15–12:45 **Speaker:** Christopher Hanusa (CUNY Queens College)

Title: Combinatorial interpretations in affine Coxeter groups of types B, C, and D

Abstract: Core partitions and abacus diagrams are two useful combinatorial objects in bijection with minimal length coset representatives in the parabolic quotient of the affine symmetric group modulo the finite symmetric group. In joint work with Brant Jones, we extend core partitions, abacus diagrams, and other combinatorial interpretations to types affine B, C, and D.

12:50–14:15 LUNCH

14:15–14:45 **Speaker:** John Enyang (University of Melbourne)

Title: A seminormal form for partition algebras

Abstract: Partition algebras were defined by Jones and Martin in the 1990s in connection with the Potts model and higher dimensional statistical mechanics. Jones showed that the partition algebras are in Schur–Weyl duality with the group algebra of the symmetric group S_n acting on the k -fold tensor power of its n -dimensional permutation representation V . In this talk we will introduce a new presentation of the partition algebras and show how this presentation is used to obtain explicit combinatorial formulae for the seminormal representations of the partition algebras. Our results generalise to the partition algebras the classical formulae given by Young for the symmetric group.

14:50–15:20 BREAK

15:20–16:20 PLENARY LECTURE

Speaker: Andrei Okounkov (Columbia University)

Title: Partitions and tableaux in high energy physics

Abstract: TBA

16:30–17:30 PLENARY LECTURE

Speaker: John Conway (Princeton University)

Title: TBA

Abstract: TBA